

**Exercise 13**

Evaluate the line integral, where  $C$  is the given curve.

$$\int_C xy e^{yz} dy, \quad C : x = t, y = t^2, z = t^3, \quad 0 \leq t \leq 1$$

**Solution**

With this parameterization in  $t$ , the line integral becomes

$$\begin{aligned} \int_C xy e^{yz} dy &= \int_0^1 x(t)y(t)e^{y(t)z(t)} \frac{dy}{dt} dt \\ &= \int_0^1 (t)(t^2)e^{(t^2)(t^3)}(2t) dt \\ &= 2 \int_0^1 t^4 e^{t^5} dt. \end{aligned}$$

Make the following substitution.

$$\begin{aligned} u &= t^5 \\ du &= 5t^4 dt \quad \rightarrow \quad \frac{du}{5} = t^4 dt \end{aligned}$$

Therefore,

$$\begin{aligned} \int_C xy e^{yz} dy &= 2 \int_{0^5}^{1^5} e^u \left( \frac{du}{5} \right) \\ &= \frac{2}{5} \int_0^1 e^u du \\ &= \frac{2}{5} (e^u) \Big|_0^1 \\ &= \frac{2}{5} (e^1 - e^0) \\ &= \frac{2}{5} (e - 1). \end{aligned}$$